Using the Dynamic Power of Microsoft Excel to Stand on the Shoulders of GIANTS

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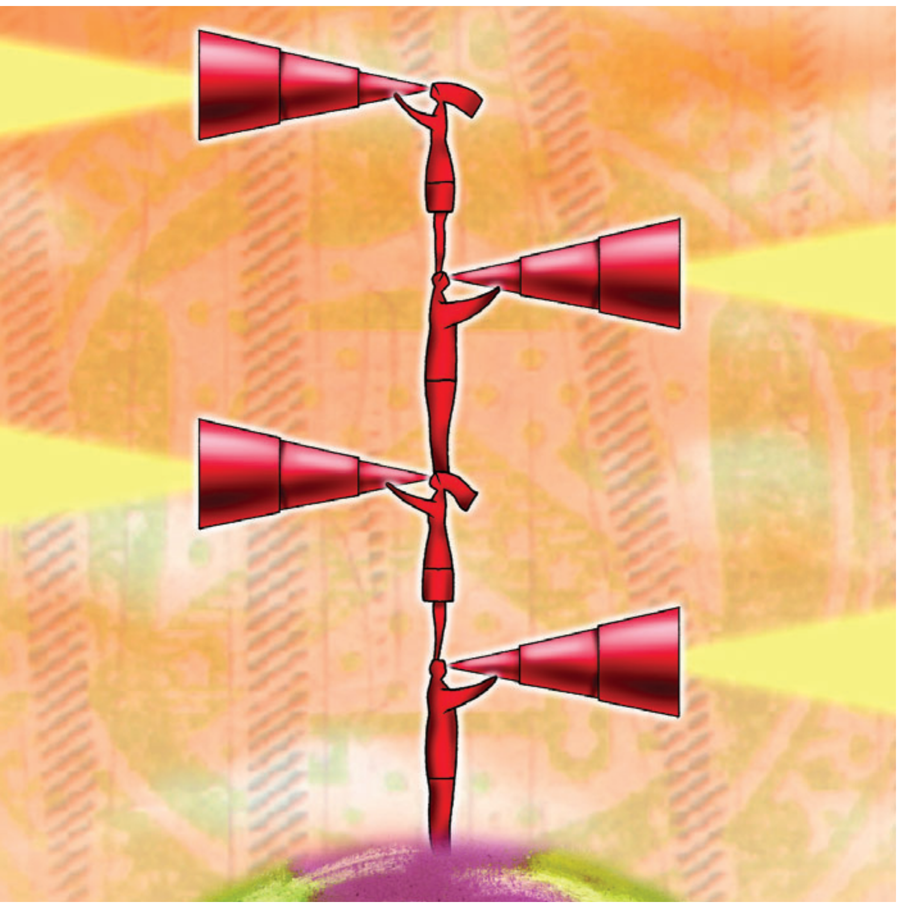


In a letter to Robert Hooke in 1676 Sir Isaac Newton wrote, "If I have seen farther than others, it is because I have stood on the shoulders of giants" (Burton 1999, p. 314). Newton was referring to his development of the calculus, which was made possible by the work of Descartes, Cavalieri, Torricelli, Wallis, Barrow, and others. Newton's comment embodies the hierarchical nature of most research in science and mathematics. Contemporary mathematicians and scientists grapple with concepts and theories that were beyond the limits of some of the greatest scientific minds in history. The knowledge that these great minds produced paves the way for present day innovations.

Students in our classes also stand on the shoulders of giants. They are asked to learn things that were widely unknown for much of recorded history. For example, the development of the real number system took centuries, yet we expect our students to learn its fundamentals in elementary school. How can this be? One reason is cultural; our modern culture relies heavily on basic mathematics. Many children are exposed to the concept of number long before they get to school, and basic understanding of real numbers is a fundamental expectation of schooling. New theories, technologies, and concepts have transformed the mathematics we teach, the manner in which we teach it, and what our students

learn. The technological revolution of the last two decades has given teachers pedagogical tools—from graphing calculators, to powerful computers running mathematical software, to the Web and its growing collection of interactive sites. These new tools allow students to explore mathematics in ways that were previously inconceivable. Harnessing this power enables students to see even further, to explore concepts, and to develop deeper understanding than is possible without modern technology.

Technology is not a priori a good thing. Giving a student a calculator, for example, does not necessarily make him or her a better mathematics student, and if used inappropriately the calculator can be a detriment to learning. Knowledge does not live in a calculator; individuals construct knowledge through their experiences. Using technology as a tool, teachers can design tasks and activities that allow students to construct deep, conceptual, mathematical knowledge. Recent software improvements turn spreadsheets into such a tool.

One advantage spreadsheets have over their graphing calculator counterparts is screen size. Tables, graphs, and equations are easily viewed and manipulated in a single window. Recent spreadsheet software is more dynamic than its predecessors. In particular, Microsoft Excel (2004) gives users the capability of making scroll bar–type sliders to control the contents of a cell. See **figure 1**. The slider that covers cells B2 and C2 controls the value in A2; clicking an arrow on the slider changes the value. (Detailed information on creating and formatting scroll bars is available online. See Buckley and Sprague 2003.) When scroll bars are used to control function parameters, users can view simultaneous changes in tables, graphs, and equations. Three tasks that make use of this interactive tool are described below. The first two tasks are generalizations of traditional algebra tasks; the third is an investigation of polynomial functions. I have used these tasks to develop preservice and in-service teachers' depth of understanding of variable and function, two concepts central to the high school curriculum. These tasks illustrate how the power of Microsoft Excel can be used to create conceptually rich learning experiences.

EXTENDING TRADITIONAL PROBLEMS

In the text *Mathematics for High School Teachers,* Usiskin, Peressini, Marchisotto, and Stanley (2003) describe a "problem analysis" as one method to investigate high school mathematics from an advanced perspective:

> Problem analysis involves more than finding different ways of solving a problem. It includes looking at a problem after it has been solved and examining what has been done. Will the method of solution work for other problems? Can we extend the problem? And so forth. (p. 5)

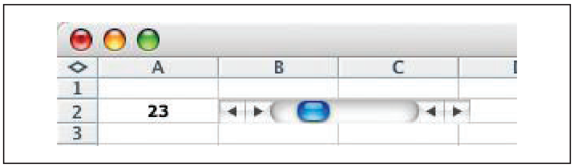


Fig. 1 Slider in Microsoft Excel

The authors use problem solving to "raise questions from the level of exercises to the level of mathematical analysis" (p. 76). One way they extend problems is by introducing parameters for values that are initially, and traditionally, constant. Generalizing problems in this manner is a way to create rich tasks for algebra and precalculus students.

Example 1. How long does it take to catch up?

Consider the following classical algebra exercise:

> Person A sets out in a car going at 50 mph. Starting 3 hours later, person B tries to catch up [along the same route]. If person B goes at 75 mph, how long does it take to catch up? (Usiskin et al. 2003, p. 76).

One approach is to let t be the time in hours that person A travels and to solve the following equation:

$$50t = 75(t - 3)$$

Thus, $t = 9$ hours. A way to generalize this problem is to vary the speeds. If person A travels at a speed of v_1 and person B at a speed of v_2 the equation becomes

$$v_1 t = v_2(t - 3).$$

For many students, solving this equation to find

$$t = \frac{3v_2}{v_2 - v_1}$$

is a meaningless exercise in algebraic manipulation, but this problem comes to life when it is considered graphically.

Figure 2 shows a Microsoft Excel worksheet illustrating the generalized version of this problem. The scroll bars control the parameters v_1 and v_2. As the user manipulates these values, the table and graphs adjust immediately. New questions about the situation can be considered that require students to make connections between the different representations: Why are the graphs linear? What happens as B's speed decreases? How are the graphs related to the equation $v_1 t = v_2(t - 3)$? Can students explain the meaning of

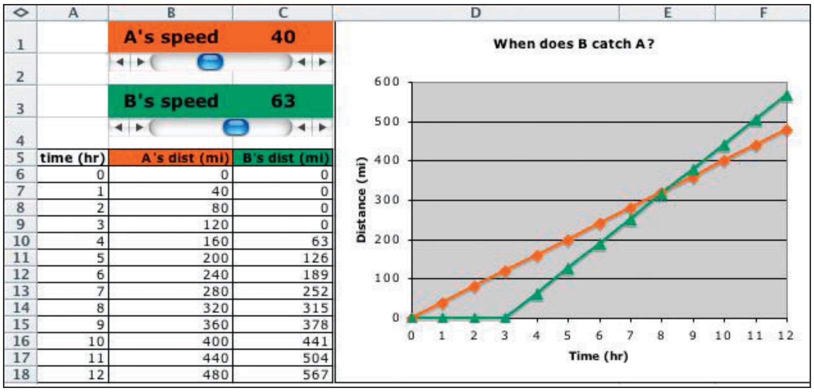


Fig. 2 Generalized car problem illustrated in Microsoft Excel

$$t = \frac{3v_2}{v_2 - v_1}$$

using the table and graph? Questions that require links between representations allow students to make meaning of the symbols that traditionally are only manipulated.

Example 2. Who will have more money?

Consider this more advanced algebra problem:

> Twin 1 and Twin 2 are planning to save for retirement. Twin 1 will save \$2000 per year from age 25 through age 34 and then no longer make annual contributions. Every year on her birthday she makes a \$2000 deposit. Twin 2 wants to "live it up" while she is still young and plans to start saving \$2000 per year when she hits age 35. Beginning on her 35th birthday she deposits \$2000 per year on the anniversary of her birth. Which twin will have more money when they retire on their 65th birthday? Assume an interest rate of 8% compounded annually.

One approach is to solve the problem algebraically, using the formulas for future value for a fixed investment (compound interest formula) and future value of an annuity to compute the twins' savings:

> *Future value of a fixed investment*—If an initial investment of P dollars is invested at an annual interest rate of r compounded once a year, the amount of money in the account after t years is
>
> $$A_F(t) = P(1 + r)^t.$$
>
> *Future value of an annuity*—If P dollars are invested at the beginning of each year for t years earning an annual interest rate of r at the end of each year, the amount of money in the account after t years is
>
> $$A_A(t) = P\left(\frac{(1+r)^{t+1} - (1+r)}{r}\right).$$

Both formulas must be used to compute the amount Twin 1 will have at retirement, but Twin 2's retirement savings can be computed directly from $A_A(t)$. Twin 1 will save

$$A_A(10) \bullet A_F(30) = \left[2000\left(\frac{(1.08)^{11} - (1.08)}{0.08}\right)\right](1.08)^{30}$$
$$= \$314{,}870.34.$$

Twin 2 will save

$$A_A(30) = 2000\left(\frac{(1.08)^{31} - (1.08)}{0.08}\right)$$
$$= \$244{,}691.74.$$

When students find this solution, they inevitably ask, "How can Twin 1 have more money for retirement when Twin 2 adds so much more money to her account?" This question can lead to several generalizations, such as, What effect does the assumed interest rate have on the result? What effect does changing the compounding period have?

I recently asked a class of preservice and in-service teachers to determine which twin will save more, for assumptions similar to those given above. Two students investigated the effects of changing the interest rate and discovered that which twin saves more depends upon the interest rate. The graphs the students produced are shown in **figure 3**. **Figure 3a** shows that when the assumed interest rate is 8 percent, Twin 1 saves approximately \$100,000 more than Twin 2. When the interest rate is 4 percent, as shown in **figure 3b**, Twin 2 saves approximately \$40,000 more than Twin 1. Experimentally they found the following:

> If the interest rate is less than or equal to 5.77 percent, Twin 2 will have more money at retirement. If the interest rate is more than or equal to 5.78 percent, Twin 1 will have more money at retirement. If the interest rate, r, is such that $5.77\% < r < 5.78\%$, then the twins will have approximately the same amount of money at retirement.

By defining parameters for the interest rate these students explored the effects of changing r, and through trial and error they found an interesting result. I do not think they would have come to such a result without the power of Microsoft Excel.

This task unrealistically assumes the interest rates are fixed and the same for each twin for the duration of the investments. These assumptions present an opportunity to discuss the roles and limitations of mathematical models. They also lead to generalizations, such as, How will the twins' savings be affected if the

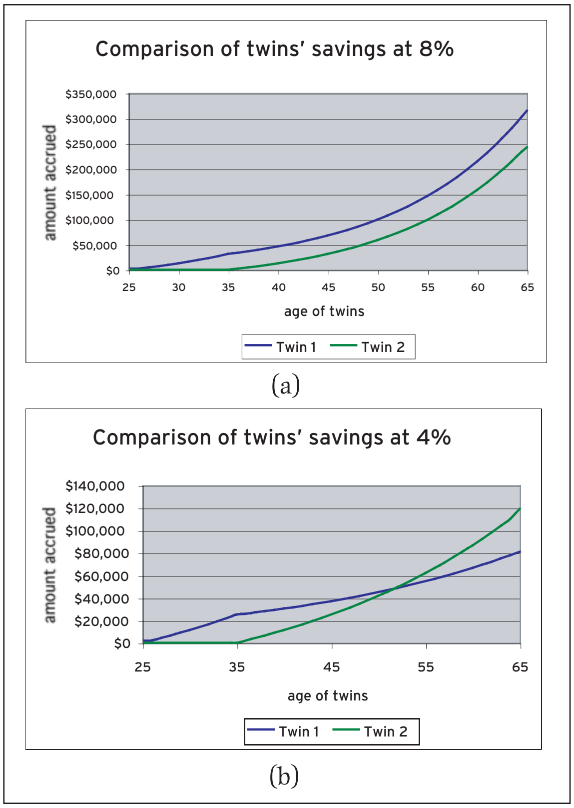


Fig. 3 A comparison of the twins' retirement, varying the interest earned

interest rate varies over the course of their lives? How does the compounding period affect the results?

INVESTIGATING FUNCTION BEHAVIOR

A traditional topic in the study of precalculus mathematics is families of functions. Students learn to distinguish polynomial types algebraically and graphically and are introduced to more in-depth analyses that include identifying intervals where functions increase and decrease, locating maxima and minima, and describing limiting behavior. Detailed study of the rate of change of a function and how that rate changes is usually left until calculus. Using finite differences, defined below, students can investigate change deeply and discover that different families of functions have different characteristics of change.

Finite Differences

In words, the first differences of a function are the differences between function values of successive integers and second differences are first differences of the first differences. Algebraically, a first difference function, *FD,* can be defined for a function *f* as

$$FD(x) = f(x+1) - f(x)$$

and a second difference function, *SD,* as

$$SD(x) = FD(x+1) - FD(x),$$

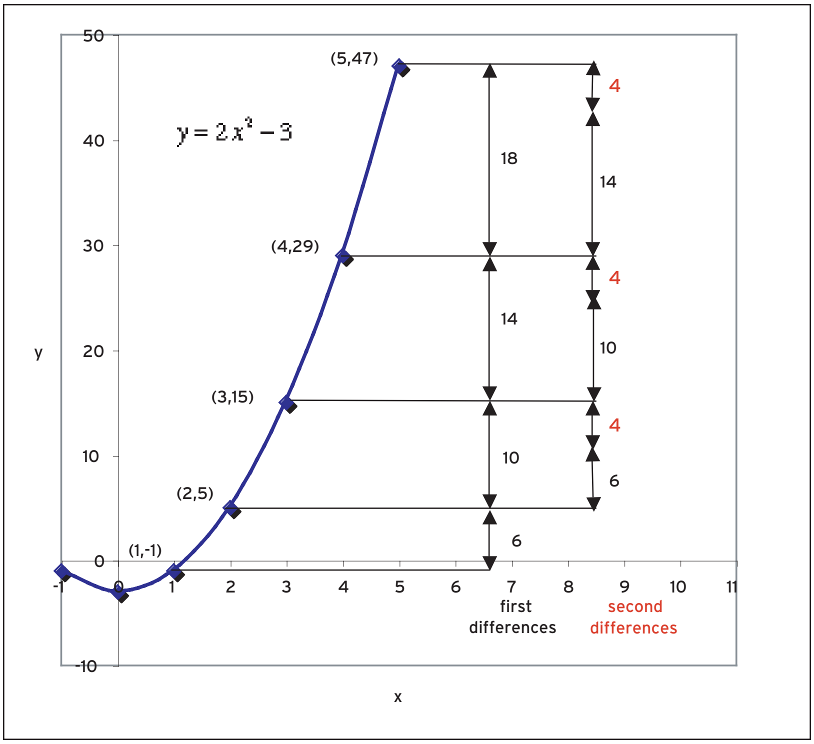


Fig. 4 Visualizing first and second differences

or equivalently

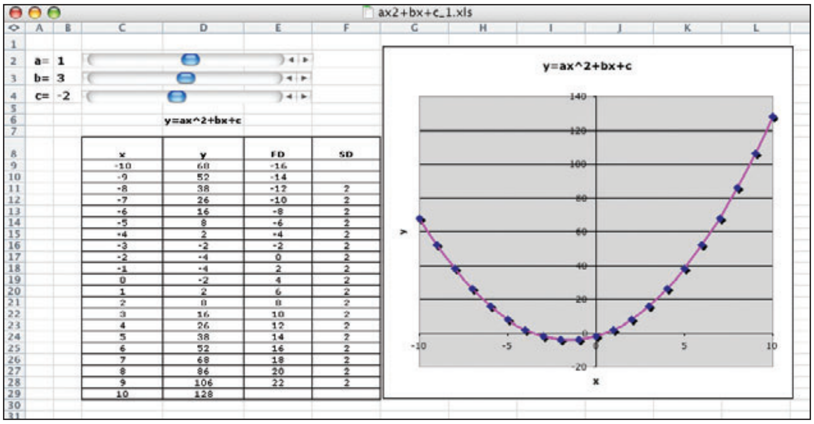
$$SD(x) = f(x+2) - 2f(x+1) + f(x).$$

Figure 4 illustrates several values of $FD(x)$ and $SD(x)$ graphically for the function *f* defined by $y = 2x^2 - 3$. The first differences (6, 10, 14, 18) come from the difference of successive ordinates. The second differences are the differences of successive first differences and have a constant value of 4. These definitions have natural extensions to third differences, fourth differences, and so on.

Finite differences provide a means to study change and are easily computed in Microsoft Excel. An investigation of quadratic functions that uses sliders and differences is described below. Before my students use first and second differences to study quadratics, I have them define these differences for linear functions and investigate the effects of variations in slope and intercept. This leads to a discussion of why the first differences are constant and equal to the slope, and why the second differences are thus 0. This activity usually leads students to attempt to generalize and to extend the pattern to other functions, in particular quadratics.

Example 3. How do a, b, and c affect $y = f(x) = ax^2 + bx + c$?

The general form of a quadratic function is $y = f(x) = ax^2 + bx + c$. For this investigation one parameter is varied while the others are held constant (e.g., $y = x^2 + c$, $y = x^2 + bx$, and $y = ax^2$). A sample spreadsheet for this



x	y	FD	SD
-10	68	-16	
-9	52	-14	
-8	38	-12	2
-7	26	-10	2
-6	16	-8	2
-5	8	-6	2
-4	2	-4	2
-3	-2	-2	2
-2	-4	0	2
-1	-4	2	2
0	-2	4	2
1	2	6	2
2	8	8	2
3	16	10	2
4	26	12	2
5	38	14	2
6	52	16	2
7	68	18	2
8	86	20	2
9	106	22	2
10	128		

Fig. 5 Sample spreadsheet for quadratic investigation

investigation is shown in **figure 5**. For each case, students consider and discuss questions such as:

- What effect does varying the parameter have on the values of the function?
- What effect does varying the parameter have on the rate of change of the function, i.e., first differences?
- What effect does varying the parameter have on the change in the rate of change, i.e., second differences?
- What changes do you observe in the graph as the parameter is varied?
- Would you expect the changes you observed when varying *c* to have similar effects if *a* and *b* were different from the fixed values you assumed for this example? Why or why not?

Students are required to refer to the table and graph in their answers. For an assessment, students write about and present summaries of their findings.

A characteristic of the polynomial family of functions is that the *n*th differences of an *n*th degree polynomial are constant, e.g., the second differences of a quadratic function are constant. This property is one of the first things students notice as they begin to vary *a, b,* and *c.* As soon as they adjust *a,* they realize the constant value of the second differences is determined by *a:* It is $2a$. Generally, they also observe a linear pattern in the first differences. Both of these observations can be proven algebraically, by computing $FD(x) = f(x+1) - f(x)$ for $y = f(x) = ax^2 + bx + c$, which shows

$$FD(x) = 2ax + a + b.$$

A similar algebraic exercise shows that $SD(x) = 2a$. Students also observe that—

- varying *c* translates the graph vertically;
- varying *b* does not change the general shape but does move the location of the vertex; and
- varying *a* controls whether the parabola opens up or down and how quickly the parabola narrows.

As an extension, students investigate whether the constant differences property holds for nonpolynomial functions. The evidence they collect both suggests that the property does not hold and yields the conjecture that constant differences are a defining property of the polynomial family.

Students in the classes in which I have used this investigation became really involved and excited. Their ability to see all of these things happening at once seemed almost liberating. Recently, for example, a student investigated differences for higher degree polynomials and noticed that the *n*th difference of an *n*th degree polynomial has the constant value $(n!)a$. Two other students observed that the path of a parabola's vertex seemed to follow a parabolic route as parameter *b* changes. Initially they were unsure how to prove that the path was parabolic until they remembered the formula for the coordinates of the vertex of a quadratic function is

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right).$$

Using this formula, the students were able to show that the path of the vertex lies on the function $f(x) = -ax^2 + c$.

CONCLUSION

In his seminal work on understanding, Richard Skemp discussed two types of understanding: instrumental and relational (1978). He defined instrumental understanding as "rules without reasons" and relational understanding as "knowing what to do and why." NCTM's *Principles and Standards for School Mathematics* (2000) promotes a vision of understanding that is consistent with Skemp's notion of relational understanding. Students not only need to know what to do, they also need to know why.

The *Principles and Standards* states four overarching goals for the study of algebra:

> Instructional programs from prekindergarten through grade 12 should enable all students to—
> - understand patterns, relations, and functions;
> - represent and analyze mathematical situations and structures using algebraic symbols;
> - use mathematical models to represent and understand quantitative relationships;
> - analyze change in various contexts. (NCTM 2000, p. 37)

Tasks like the ones described above that make use of parameters help students attain these goals.

One of Microsoft Excel's strengths is its ability to integrate multiple representations. To achieve relational understanding in calculus and beyond, students need to have a flexible understanding of functions, that is, they need to be able to move between different representations of functions as appropriate. Using Microsoft Excel in the manner described in this article forces students to think of the same phenomena in multiple ways and to make mathematical connections. This focus on connections will increase students' flexibility.

These tasks also contain a very important, but more subtle, conceptual focus. Students are forced to work with different conceptions of variables. For example, in the car problem t is an unknown quantity initially, a single value that solves the equation

$$50t = 75(t - 3).$$

In the generalized car problem, t is a varying quantity, the independent variable for the two distance functions. Variables must also be conceptualized as parameters as in the case of v_1 and v_2 in the car example and r in the interest examples. The process of creating the spreadsheet forces students to construct variable equations to represent the relationships. Research has shown that the transition between different conceptions of variable is an area of difficulty as the concept of variable develops in students (Kuchemann 1981). Developing the concept of variable is an important, but often overlooked, component of the study of algebra. By completing tasks like the ones above, students' continued use of different conceptions of variable should deepen their understanding of this essential idea.

In my growth as a teacher of mathematics, I have found that using technology has helped move my classroom from teacher-centered to student-centered. Through problem generalizations and investigations, students communicate mathematics to each other, to the entire class, and in writing. Lively classroom discussions lead to conjectures, proofs, and further investigations. All of this takes place in the context of meaningful learning of important mathematics.

A nice outcome of introducing preservice and in-service teachers to this tool was that they really enjoyed using it. The day after students worked on the parabola activity, a student sent me his spreadsheet and commented, "I fooled around with this a little more just 'cause it's fun." Later in the day he sent another e-mail with a spreadsheet he had created to investigate the behavior of $y = e^{ax} \cdot b \sin(cx)$ and said, "You don't know what you've started." Two other creative uses for the sliders came from the two in-service teachers in the class. One teacher created a spreadsheet investigation for trigonometric functions. He was able to project the image of his spreadsheet and allow students to manipulate the parameters with a wireless mouse. The second teacher created a spreadsheet with sliders to illustrate the effects of benefits on teachers' salaries. This spreadsheet was used during contract negotiations to get immediate feedback on proposals. These examples offer evidence of how empowering this tool can be.

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Most of the students we will have in our classrooms will never be the research mathematicians that Sir Isaac Newton was, extending the boundaries of known mathematics, but as they see mathematics in a new way and discover things that are new to them, they "stand on the shoulders" of rich tasks and powerful tools.

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